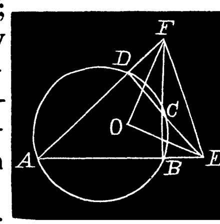
Show that the bisectors of the angles formed by producing the sides of an inscribed quadrilateral intersect each other at right angles.

Solution by the Proposer.

Let $ABCD$ be the inscribed quadrilateral and FO and EO the bisectors of the angles F and E, respectively, formed by producing the sides of the quadrilateral. Denote the angle EAF by A; AFB, by F; BFE, by F'; AED, by E; DEF, by E'; FCE,$=DCB$, by C; and FOE, by O. Then $A+C=2$ rt. angles..(1); being opposite angles of an inscribed quadrilateral. Also, in the triangle AFE, $A+F+F'+E+E'=2$ rt. angles..(2); in the triangle FOE, $\frac{1}{2}F+F'+\frac{1}{2}E+E'+O=2$ rt. angles....(3); and, in the triangle FCE, $F'+E'+C=2$ rt. angles....(4).



Multiplying (3) by two and subtracting (4) from the resulting equation, we have $F+F'+E+E'+2O-C=2$ rt. angles..(5). Subtracting (5) from (2), we have $A+C-2O=0$, whence $2O=A+C=2$ rt. angles. $\therefore$ $O=$a rt. angle.

Q. E. D.

PROBLEMS.

2. Show that $\frac{1}{2}\pi=\left[\frac{2.\ 4.\ 6.\ 8.\ 10}{1.\ 3.\ 5.\ 7.\ 9}\cdots\right]^2$, *Wallis's* expression for π.

[Selected from *Bowser's Trigonometry.*]

3. If A be the area of the circle inscribed in a triangle, A_1, A_2, A_3 the areas of the escribed circles, show that $\frac{1}{\sqrt{A}}=\frac{1}{\sqrt{A_1}}+\frac{1}{\sqrt{A_2}}+\frac{1}{\sqrt{A_3}}$.

[Selected from *Todhunter's Plane Trigonometry.*]

4. Three circles whose radii are a, b, and c touch each other externally; prove that the tangents at the points of contact meet in a point whose distance from any one of them is $\left(\frac{abc}{a+b+c}\right)^{\frac{1}{2}}$ [Selected from *Todhunter's Plane Trigonometry.*]

5. **Proposed by ADOLPH BAILOFF, Durand, Wisconsin.**

If from a varible point in the base of an isosceles triangle, perpendiculars are drawn to the sides, the sum of the perpendicular is constant and equal to the perpendicular let fall from either extremity of the base to the opposite side.

6. **Proposed by EARL D. WEST, West Middleburg, Logan county, Ohio.**

Having given the sides 6, 4, 5, and 3 respectively of a trapezium, inscribed in a circle, to find the diameter of the circle.

7. **Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in the Ohio University, Athens, Ohio.**

Through each point of the straight line $x=my+h$ is drawn a chord of the parabola $y^2=4ax$, which is bisected in the point. Prove that this chord touches the parabola $(y-2mn)^2=8a(x-h)$.

8. **Proposed by ADOLPH BAILOFF, Durand, Wisconsin.**

If the two exterior angles at the base of a triangle are equal, the triangle is isosceles.

9. Proposed by J. C. GREGG, Brazil, Indiana.

Two circles intersect in A and B. Through A two lines CAE and DAF are drawn, each passing through a centre and terminated by the circumferences. Show that $CA\times AE=DA\times AF$. [*Euclid.*]

10. Proposed by ERIC DOOLITTLE, Instructor in Mathematics, State University of Iowa, Iowa City.

If MN be any plane, and A and B any point without the plane, to find a point P, in the plane, such that $AP+PB$ shall be a minimum.

11. Proposed by Miss LEOTA MILLER, B. L., Professor of Natural Science and Art, Kidder Institute, Kidder Missouri.

A gentleman's residence is at the center of his circular farm containing $a=900$ acres. He gives to each of his $m=7$ children an equal circular farm as large as can be made within the original farm; and he retains as large a circular farm of which his residence is the center, as can be made after the distribution. Required the area of the farms made.

12. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Let OA and OB represent two variable conjugate semi-diameters of the ellipse $\frac{y^2}{a^2}+\frac{y^2}{b^2}=1$. On the chord AB as a side describe an equilateral triangle ABC. Find the locus of C.

13. Proposed by HENRY HEATON, M. S., Atlantic, Iowa.

Through two given points to pass four spherical surfaces tangent to two given spheres.

14. Proposed by HENRY HEATON, M. S., Atlantic City, Iowa.

Through a given point to draw four circles tangent to two given circles.

15. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

A man starts from the centre of a circular 10 acre field and walks due north a certain distance, then turns and walks south-west till he comes to the circumference, walking altogether 40 rods. How far did he walk before making the turn?

16. Proposed by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School. Philadelphia, Pennsylvania.

Three lights of intensities 2, 4 and 5 are placed respectively at points the coordinates of which are (0,3) (4,5) and (9,0). Find a point in the plane of the lights equally illuminated by all of them.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

PROBLEMS.

1. Find the moment of inertia about the origin, of the area included within the parabola $y^2=4ax$, the line $x+y=4a$, and the axis of x.

[Selected from *Osborne's Differential and Integral Calculus.*]